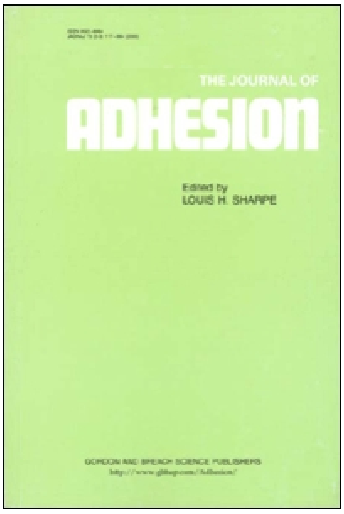

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A Stress Analysis of Butt Adhesive Joints Under Torsional Loads

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The stress distribution and the displacement are examined when a butt adhesive joint, in which two dissimilar tubular shafts are joined, is subjected to a torsional load. In the analyses, general representations of the stresses and the displacements are given as a torsion problem when two dissimilar tubular shafts are band-adhesively bonded. Next, in the case of shafts with the same material, effects of the ratio of the shear modulus of an adhesive to that of shafts, the thickness of the adhesive and the bonded position of band-adhesive on the stress distribution are made clear by numerical computations. Moreover, when solid shafts are joined, these effects are made clear by the similar analyses and numerical computations.

KEY WORDS Butt joint; elasticity; stress analysis; shear modulus; strength design; torsional load.

1. INTRODUCTION

With the advance in adhesive materials for structural use, the assembly of machine elements using an adhesive has recently been investigated as a new joining method.[1,2] In the field of machine tools, demands for higher accuracy, stiffness and lightening of the tools are increasing, so that new manufacturing systems such as computer aided manufacturing (CAM) and flexible manufacturing system (FMS) can be employed. In addition, new industrial

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materials such as ceramics and composite materials will be used more widely in the future. Under the existing conditions mentioned above, adhesive joining, in addition to the conventional joining techniques such as bolting and riveting, becomes useful for the reasons that it is possible to join different materials easily, to lighten joined structures and to manufacture an accurate machine tool more easily by leaving the shape accuracy of it to an adhesive layer in an assembly.

Up to now, however, the strength design of an adhesive joint has not been established and data for design are markedly fewer as compared with those for conventional joints. Consequently, mechanical structures with adhesive joints are not always used with full confidence. In fact, such joints are often used as an aid in conventional joining.

Some experimental and theoretical studies have been done on the stress distributions and the displacements of an adhesive joint subjected to external loads such as tension, bending and shearing.[3–6] On the other hand, studies are few on the stress distributions and the displacements of joints subjected to torsional loads.[7]

The purpose of this study is to obtain fundamental data for the strength design of an adhesively joined transmitting shaft which is assembled in machine tools or other mechanical structures and is subjected to a torsional load. In this study, the stress distributions and the displacements of a butt adhesive joint, in which two dissimilar tubular shafts are joined, are examined as a torsion problem.

Next, in consideration of imperfect adhesive bonding, when the shafts with the same material and the same dimension are bonded with a band of adhesive, *i.e.* partially bonded, effects of the ratio of the shear modulus of the adhesive to that of the shafts, the thickness of the adhesive and the position of the band of adhesive on the stress distributions and the displacements are made clear by numerical analyses.

2. THEORETICAL ANALYSIS

Figure 1 shows a model for analysis of a butt adhesive joint. Two semi-infinite dissimilar tubular shafts, called adherend(II) and adherend (III) hereinafter, are joined by the adhesive(I) and the

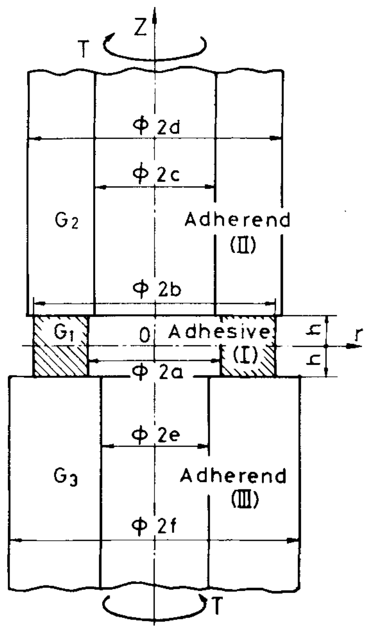

FIGURE 1 Model of adhesive joint of shafts subjected to a torsional load.

joint is subjected to a torsional load T at the ends. The inner and the outer diameters of the adherend (II) and (III) are designated by 2c, 2d, 2e and 2f, the inner and the outer diameters of the adhesive band (I) are designated by 2a and 2b respectively. Their shear moduli and Poisson's ratios are G_2, ν_2, G_3, ν_3, G_1 and ν_1, respectively.

By using cylindrical co-ordinates (r, θ, z), the stresses and the displacement are expressed as follows:

$$\frac{\partial^2 V_\theta}{\partial r^2}+\frac{1}{r}\frac{\partial V_\theta}{\partial r}-\frac{V_\theta}{r^2}+\frac{\partial^2 V_\theta}{\partial z^2}=0 \tag{1}$$

$$\tau_{\theta z}=G\frac{\partial V_\theta}{\partial z} \tag{2}$$

$$\tau_{r\theta}=Gr\frac{\partial}{\partial r}\left(\frac{V_\theta}{r}\right) \tag{3}$$

In these equations, V_θ denotes the displacement in the circumferential direction and $\tau_{\theta z}$ and $\tau_{r\theta}$ denote the shear stresses.

Boundary conditions of the joint are expressed by the following equations (4)–(13).

$$(V_{\theta 1})_{z=h} = (V_{\theta 2})_{z=h} \qquad (a \leqq r \leqq b) \tag{4}$$

$$(V_{\theta 1})_{z=-h} = (V_{\theta 3})_{z=-h} \qquad (a \leqq r \leqq b) \tag{5}$$

$$(\tau_{\theta z1})_{z=h} = (\tau_{\theta z2})_{z=h} \qquad (a \leqq r \leqq b) \tag{6}$$

$$(\tau_{\theta z1})_{z=-h} = (\tau_{\theta z3})_{z=-h} \qquad (a \leqq r \leqq b) \tag{7}$$

$$(\tau_{\theta z2})_{z=h} = 0 \qquad (c \leqq r < a), \qquad (b < r \leqq d) \tag{8}$$

$$(\tau_{\theta z3})_{z=-h} = 0 \qquad (e \leqq r < a), \qquad (b < r \leqq f) \tag{9}$$

$$(\tau_{r\theta 1})_{r=a} = (\tau_{r\theta 1})_{r=b} = 0 \qquad (|z| \leqq h) \tag{10}$$

$$(\tau_{r\theta 2})_{r=c} = (\tau_{r\theta 2})_{r=d} = 0 \qquad (z > h) \tag{11}$$

$$(\tau_{r\theta 3})_{r=e} = (\tau_{r\theta 3})_{r=f} = 0 \qquad (z < -h) \tag{12}$$

$$\text{Torsional load } T = \text{const.} \qquad (|z| \leqq \infty) \tag{13}$$

The subscripts 1, 2 and 3 correspond to the adherend (I), adhesive (II) and adherend(III), respectively.

$V_{\theta 1}$ is solved by a method of separation of variables taking Eqs. (10)–(13) into consideration and then the shear stresses $\tau_{\theta z1}$ and $\tau_{r\theta 1}$ of the adhesive(I) are expressed from Eqs. (2) and (3) as follows;

$$G_1 V_{\theta 1} = A_0' r + A_0 zr + \sum_{n=1}^{\infty} C_1(\alpha_n r)[A_n \sinh(\alpha_n z) + B_n \cosh(\alpha_n z)] \tag{14}$$

$$\tau_{\theta z1} = A_0 r + \sum_{n=1}^{\infty} \alpha_n C_1(\alpha_n r)[A_n \cosh(\alpha_n z) + B_n \sinh(\alpha_n z)] \tag{15}$$

$$\tau_{r\theta 1} = -\sum_{n=1}^{\infty} \alpha_n C_2(\alpha_n r)[A_n \sinh(\alpha_n z) + B_n \cosh(\alpha_n z)] \tag{16}$$

where,

$$C_j(\alpha_n r) = J_j(\alpha_n r) - J_2(\alpha_n a) Y_j(\alpha_n r)/Y_2(\alpha_n a)$$

$J_j(\alpha_n r)$: Bessel function of 1st kind of order j

$Y_j(\alpha_n r)$: Bessel function of 2nd kind of order j

α_n: the n th positive root satisfying $C_2(\alpha_n b) = 0$

and $C_2(\alpha_n a)$ becomes 0 too, so that Eq.(10) is satisfied under these definitions.

Similarly, Eqs. (17)–(19) for the adherend(II) and Eqs. (20)–(22) for the adherend(III), are expressed.

$$G_2 V_{\theta 2} = D_0' r + D_0 zr + \sum_{m=1}^{\infty} D_m C_1(\beta_m r)\exp(-\beta_m z) \tag{17}$$

$$\tau_{\theta z2} = D_0 r - \sum_{m=1}^{\infty} D_m \beta_m C_1(\beta_m r)\exp(-\beta_m z) \tag{18}$$

$$\tau_{r\theta 2} = -\sum_{m=1}^{\infty} D_m \beta_m C_2(\beta_m r)\exp(-\beta_m z) \tag{19}$$

$$G_3 V_{\theta 3} = F_0' r + F_0 zr + \sum_{l=1}^{\infty} F_l C_1(\gamma_l r)\exp(\gamma_l z) \tag{20}$$

$$\tau_{\theta z3} = F_0 r + \sum_{l=1}^{\infty} F_l \gamma_l C_1(\gamma_l r)\exp(\gamma_l z) \tag{21}$$

$$\tau_{r\theta 3} = -\sum_{l=1}^{\infty} F_l \gamma_l C_2(\gamma_l r)\exp(\gamma_l z) \tag{22}$$

β_m and γ_l are the m th and l th positive roots satisfying $C_2(\beta_m b) = 0$ and $C_2(\gamma_l b) = 0$, respectively and $C_2(\beta_m a)$ and $C_2(\gamma_l a)$ become 0 too.

Thus Eqs. (11) and (12) are satisfied. A_0', A_0, $\{A_n\}$, $\{B_n\}$, D_0', D_0, $\{D_m\}$, F_0', F_0 and $\{F_l\}$ in Eqs. (14)–(22) are undetermined coefficients obtained from the other boundary conditions. Eq. (13), concerning with the torsional load T, is written as Eq. (23), then D_0 and F_0 are determined as Eqs. (24) and (25).

$$T = \int_c^d (\tau_{\theta z2})_{z\to\infty} \times 2\pi r^2\, dr = \int_e^f (\tau_{\theta z3})_{z\to -\infty} \times 2\pi r^2\, dr \tag{23}$$

$$D_0 = \frac{2T}{\pi(d^4 - c^4)} \tag{24}$$

$$F_0 = \frac{2T}{\pi(f^4 - e^4)} \tag{25}$$

From Eq. (4), concerned with the displacement V_θ at the interface between the adhesive (I) and the adherend (II), Eq. (26) is obtained.

$$A_0'r + A_0hr + \sum_{n=1}^{\infty} C_1(\alpha_n r)[A_n \sinh(\alpha_n h) + B_n \cosh(\alpha_n h)]$$
$$= \frac{G_1}{G_2}\left[D_0'r + D_0hr + \sum_{m=1}^{\infty} D_m C_1(\beta_m r)\exp(-\beta_m h)\right] \quad (a \leqq r \leqq b) \tag{26}$$

Transforming the right side of Eq. (26) in the region $a \leqq r \leqq b$ into the expansions of Bessel functions by terms of $C_1(\alpha_n r)$ and comparing with the left side of the equation, the first member is expressed as follows;

$$\frac{b^4 - a^4}{4}\left[(A_0' + A_0h) - \frac{G_1}{G_2}(D_0' + D_0h)\right]$$
$$= \frac{G_1}{G_2}\sum_{m=1}^{\infty}\frac{D_m}{\beta_m}[b^2C_2(\beta_m b) - a^2C_2(\beta_m a)]\exp(-\beta_m h) \tag{27}$$

and the second member is as follows;

$$\frac{1}{2}[A_n \sinh(\alpha_n h) + B_n \cosh(\alpha_n h)] \times [b^2C_1^2(\alpha_n b) - a^2C_1^2(\alpha_n a)]$$
$$= \frac{G_1}{G_2}\sum_{m=1}^{\infty}\frac{D_m\beta_m \exp(-\beta_m h)}{\beta_m^2 - \alpha_n^2}$$
$$\times [bC_2(\beta_m b)C_1(\alpha_n b) - aC_2(\beta_m a)C_1(\alpha_n a)] \tag{28}$$

Similarly, Eqs. (29)–(31) are obtained from Eq. (5) concerned with the displacement V_θ at the interface between the adhesive(I) and the adherend(III).

$$A_0'r - A_0hr - \sum_{n=1}^{\infty} C_1(\alpha_n r)[A_n \sinh(\alpha_n h) - B_n \cosh(\alpha_n h)]$$
$$= \frac{G_1}{G_3}\left[F_0'r - F_0hr + \sum_{l=1}^{\infty} F_l C_2(\gamma_l r)\exp(-\gamma_l h)\right] \quad (a \leqq r \leqq b) \tag{29}$$

$$\frac{b^4 - a^4}{4}\left[(A_0' - A_0h) - \frac{G_1}{G_3}(F_0' - F_0h)\right]$$
$$= \frac{G_1}{G_3}\sum_{l=1}^{\infty}\frac{F_l}{\gamma_l}[b^2C_2(\gamma_l b) - a^2C_2(\gamma_l a)]\exp(-\gamma_l h) \tag{30}$$

$$-\frac{1}{2}[A_n \sinh(\alpha_n h) - B_n \cosh(\alpha_n h)] \times [b^2 C_1^2(\alpha_n b) - a^2 C_1^2(\alpha_n a)]$$

$$= \frac{G_1}{G_3} \sum_{l=1}^{\infty} \frac{F_l \gamma_l \exp(-\gamma_l h)}{\gamma_l^2 - \alpha_n^2} [bC_2(\gamma_1 b) C_1(\alpha_n b) - aC_2(\gamma_l a) C_1(\alpha_n a)] \quad (31)$$

From Eqs. (6) and (8), concerning the shear stress $\tau_{\theta z}$ at the interface between the adhesive (I) and the adherend (II), Eq. (32) is obtained.

$$D_0 r - \sum_{n=1}^{\infty} D_m \beta_m C_1(\beta_m r) \exp(-\beta_m h)$$

$$= \begin{cases} A_0 r + \sum_{n=1}^{\infty} \alpha_n C_1(\alpha_n r)[A_n \cosh(\alpha_n h) + B_n \sinh(\alpha_n h)] & (a \leqq r \leqq b) \\ 0 & (c \leqq r < a), \quad (b < r \leqq d) \end{cases} \quad (32)$$

Transforming the right side of Eq. (32) in the region $c \leqq r \leqq d$ into the expansions of Bessel functions by terms of $C_1(\beta_m r)$ and comparing with the left side of the equation, the following equations are obtained.

$$D_0 = A_0 \frac{b^4 - a^4}{d^4 - c^4} \quad (33)$$

$$D_m \beta_m \exp(-\beta_m h)$$
$$= \frac{2}{d^2 C_1^2(\beta_m d) - c^2 C_1^2(\beta_m c)} \left\{ \frac{A_0}{\beta_m} [a^2 C_2(\beta_m a) - b^2 C_2(\beta_m b)] \right.$$
$$+ \sum_{n=1}^{\infty} \frac{\alpha_n \beta_m}{\alpha_n^2 - \beta_m^2} [A_n \cosh(\alpha_n h) + B_n \sinh(\alpha_n h)]$$
$$\left. \times [bC_2(\beta_m b) C_1(\alpha_n b) - aC_2(\beta_m a) C_1(\alpha_n a)] \right\} \quad (34)$$

Substituting Eq. (33) into Eq. (24), Eq. (35) is obtained.

$$A_0 = \frac{2T}{\pi(b^4 - a^4)} \quad (35)$$

Similarly, from Eqs. (7) and (9) concerning the shear stress $\tau_{\theta z}$ at the interface between the adhesive (I) and the adherend (III), Eq. (36) is obtained.

$$F_0 r - \sum_{l=1}^{\infty} F_l \gamma_l C_l(\gamma_l r)\exp(-\gamma_l h)$$

$$= \begin{cases} A_0 r + \sum_{n=1}^{\infty} \alpha_n C_1(\alpha_n r)[A_n \cosh(\alpha_n h) - B_n \sinh(\alpha_n h)] & (a \leqq r \leqq b) \\ 0 \qquad (e \leqq r < a), & (b < r \leqq f) \end{cases} \tag{36}$$

and

$$F_l \gamma_l \exp(-\gamma_l h) = \frac{2}{f^2 C_1^2(\gamma_l f) - e^2 C_1^2(\gamma_l e)} \left\{ \frac{A_0}{\gamma_l} [b^2 C_2(\gamma_l b) - a^2 C_2(\gamma_l a)] \right.$$

$$+ \sum_{n=1}^{\infty} \frac{\alpha_n \gamma_l}{\alpha_n^2 - \gamma_l^2} [A_n \cosh(\alpha_n h) - B_n \sinh(\alpha_n h)]$$

$$\left. \times (a C_2(\gamma_l a) C_1(\alpha_n a) - b C_2(\gamma_l b) C_1(\alpha_n b)) \right\} \tag{37}$$

Undetermined coefficients A_0, D_0 and F_0 are determined from Eqs. (35), (24) and (25), respectively and $\{A_n\}$, $\{B_n\}$, $\{D_m\}$ and $\{F_l\}$ are determined by solving four infinite simultaneous equations (28), (31), (34) and (37). Besides, A_0', D_0' and F_0' cannot be determined independently but they are concerned with the displacement V_θ only. So they are determined in this analysis such that the displacement becomes 0 at the outer circumference of the adherends. Using these determined coefficients, the stresses and the displacements are obtained.

3. NUMERICAL RESULTS

3.1 Band-adhesive Joint of Tubular Shafts

Numerical computations are carried out in the case where the two adherends (II) and (III) are the same material ($G_2 = G_3$) and of the

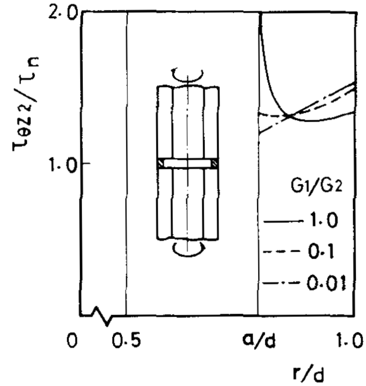

FIGURE 2 Effects of the ratio of shear modulus of adherends to that of an adhesive on the shear stress distribution $\tau_{\theta z2}/\tau_n$, type(A): $(2h/d = 0.1,\ z = h,\ a = (c^2 + d^2)^{\frac{1}{2}}/(2)^{\frac{1}{2}},\ c = d/2,\ b = d)$.

same dimensions ($c = e,\ d = f,\ c/d = 0.5$). In order to investigate the effect of the position of the adhesive band on the stress distributions and the displacements, two types of adhesive joints are examined numerically, that is, one is bonded at the outer interface, called type (A) and the other is bonded at the inner interface, called type (B), where the bonded area of each type is the same. Moreover, the effect of the ratio of shear modulus of the adhesive to that of the adherend and the effect of the thickness of the adhesive are examined in the case of the two types mentioned

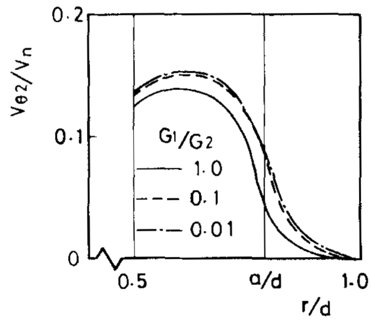

FIGURE 3 Effects of the ratio of shear modulus of adherends to that of an adhesive on the displacement $V_{\theta 2}/V_n$, type(A): $(2h/d = 0.1,\ z = h,\ a = (c^2 + d^2)^{\frac{1}{2}}/(2)^{\frac{1}{2}},\ c = d/2,\ b = d)$.

above. In computations, the number of terms, N, of the series is taken as 200, which is checked to obtain the stresses and the displacements in satisfactory accuracy.

Figures 2 and 3 show the effect of the ratio G_1/G_2 of the shear modulus of the adhesive to that of the adherend on the shear stresses and the displacement at the interface between the adhesive and the adherend, respectively, in the case of type (A). Also, these effects in the case of type (B) are shown in Figures 4 and 5. From these figures, the maximum stresses become large with an increase of the ratio G_1/G_2 at the inner circumference of the interface, *i.e.* $r = a$, in the case of type (A) and at the outer circumference of the interface, *i.e.* $r = b$, in the case of type (B). On the other hand, the maximum displacements become small with an increase of the ratio G_1/G_2 in both cases. Moreover, comparing the results of type (A) with those of type (B), the maximum stresses and the displacements become smaller in the case of type (A), *i.e.* the case where tubular shafts are bonded at the outer interface of the adherends.

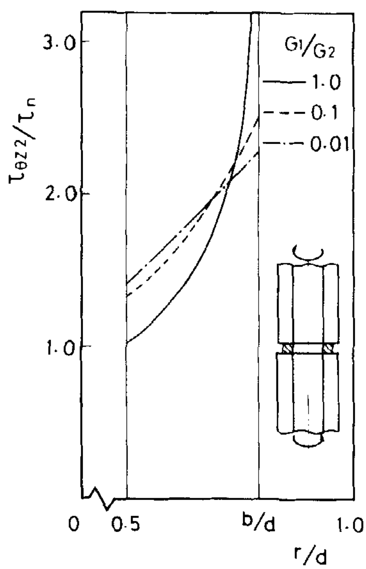

FIGURE 4 Effects of the ratio of shear modulus of adherends to that of an adhesive on the shear stress distribution $\tau_{\theta z2}/\tau_n$, type(B): $(2h/d = 0.1,\ z = h,\ b = (c^2 + d^2)^{\frac{1}{2}}/(2)^{\frac{1}{2}},\ c = d/2,\ a = c)$.

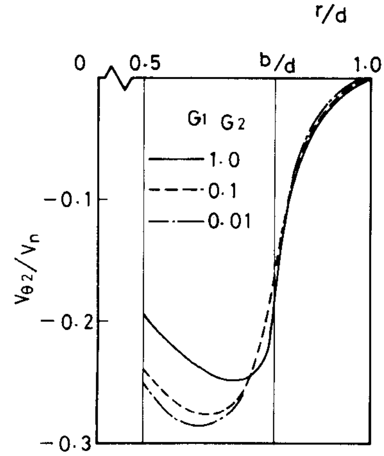

FIGURE 5 Effects of the ratio of shear modulus of adherends to that of an adhesive on the displacement $V_{\theta 2}/V_n$, type(B): $(2h/d = 0.1,\ z = h,\ b = (c^2 + d^2)^{\frac{1}{2}}/(2)^{\frac{1}{2}},\ c = d/2,\ a = c)$.

Figure 6 shows the effect of the thickness of the adhesive on the stress distributions in the case of type (A). From this figure, it is seen that the singularity of the stress becomes larger at the inner circumference, *i.e.* $r = a$, with a decrease of the thickness $2h$.

The maximum shear stress is 1.0 at the outer circumference of the

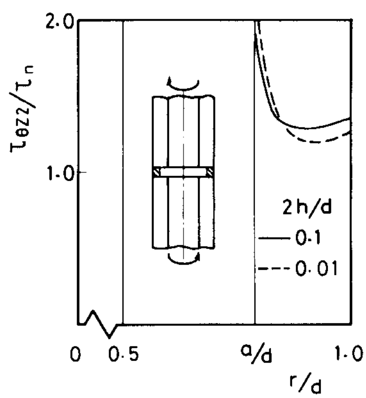

FIGURE 6 Effects of the thickness of adhesive on the shear stress distribution $\tau_{\theta z2}/\tau_n$, type(A): $(2h/d = 0.1,\ z = h,\ a = (c^2 + d^2)^{\frac{1}{2}}/(2)^{\frac{1}{2}},\ c = d/2,\ b = d)$.

interface which is completely bonded, whereas it becomes larger, that is, about 1.5 in the band-adhesive joint of type (A) when the ratio G_1/G_2 of shear modulus is 0.01, as shown in Figure 2.

3.2 Band-adhesive Joint of Solid Shafts

Numerical computations are also carried out for the band-adhesive joint, in which two solid shafts with the same materials and the same dimensions (the outer diameter is denoted by d) are joined, as a special case.

Figures 7 and 8 show the effects of the ratio G_1/G_2 of shear modulus on the shear stress distributions and on the displacements in the case of type (A), that is, for two solid shafts bonded at the outer interface. From these figures, it is seen that both the stress and the displacement distributions are similar to the results shown in Figures 2 and 4 of the adhesive joint with the tubular shafts. In addition, it is made clear that the maximum stress and the displacement in the case of type (A) become smaller than those in the case of type (B).

The maximum shear stress is 1.0 at the outer circumference of the interface which is completely bonded, whereas it becomes larger, that is, about 1.3 in the band-adhesive joint of type (A) when the ratio G_1/G_2 of shear modulus is 0.01, as shown in Figure 7.

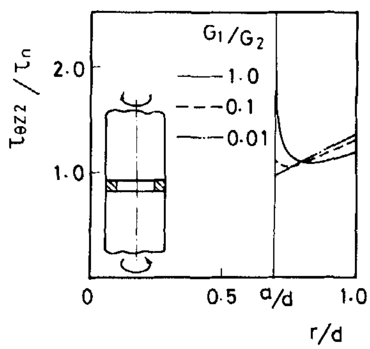

FIGURE 7 Effects of the ratio of shear modulus of adherends to that of an adhesive on the shear stress distribution $\tau_{\theta z2}/\tau_n$, type(A); ($2h/d = 0.1$, $z = h$, $a = d/(2)^{\frac{1}{2}}$, $b = d$).

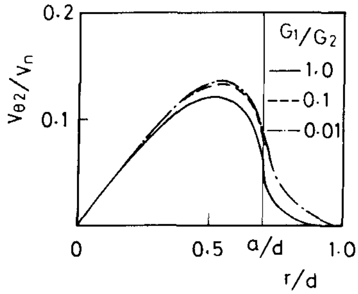

FIGURE 8 Effects of the ratio of shear modulus of adherends to that of an adhesive on the displacement $V_{\theta 2}/V_n$, type(A): $(2h/d = 0.1,\ z = h,\ a = d/(2)^{\frac{1}{2}},\ b = d)$.

4. CONCLUSIONS

The present paper deals with a stress analysis of a butt adhesive joint subjected to a torsional load and discussion is made on some mechanical characteristics of the joint from theoretical analyses.

The results obtained are as follows;

1) The larger the ratio of the shear modulus of an adhesive to that of the adherends or the smaller the thickness of the adhesive becomes, the larger the singularity of the stress becomes at the inner and the outer circumferences of the interface

2) The torsional strength of the band-adhesive joint in which tubular shafts are bonded at the outer interface of the adherends is larger than that for tubular shafts bonded at the inner interface.

3) The maximum shear stress of the joint which is bonded at the outer interface of the tubular shafts increases by 50% compared with the joint in which the interfaces are completely bonded. In the case of solid shafts, it increases by 30%, when the ratio of shear modulus of the adhesive to that of the adherends is smaller.

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